Systems with superabsorbing states

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We report on some extensive analyses of a recently proposed model [A. Lipowski, Phys. Rev. E **60**, 6255 (1999)] with infinitely many absorbing states. By performing extensive Monte Carlo simulations, we have determined critical exponents and shown strong evidence that this model is not in the directed percolation universality class. The conjecture that this two-dimensional model exhibits a dimensional reduction (behaving as one-dimensional directed percolation) is firmly disproven. The reason for the model not exhibiting standard directed percolation scaling behavior is traced back to the existence of what we call *superabsorbing sites*, i.e., absorbing sites that cannot be directly activated by the presence of neighboring activity in one or more than one direction. Supporting this claim we present two strong evidences: (i) in one dimension, where superabsorbing sites do not appear at the critical point, the system behaves as directed percolation, and (ii) in a modified two-dimensional variation of the model, defined on a honeycomb lattice, for which superabsorbing sites are very rarely observed, directed percolation behavior is recovered. Finally, a parallel updating version of the model exhibiting a nonequilibrium first-order transition is also reported.

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I. INTRODUCTION

Phase transitions separating active from fluctuation-free absorbing phases appear in a vast group of physical phenomena and models as, for instance, directed percolation [1,2], catalytic reactions [3], the pining of surfaces by disorder [4], the contact process [5], damage spreading transitions [6], nonequilibrium wetting [7], or sandpiles [8,9] See [1] and [2] for recent reviews. Classifying these transitions into universality classes is a first priority theoretical task. As conjectured by Grassberger and Janssen [10] some time ago and corroborated by a very large number of theoretical studies and computer simulations, systems exhibiting a continuous transition into a unique absorbing state with no extra symmetry or conservation law belong to one and the same universality class, namely, that of directed percolation (DP). At a field theoretical level this class is represented by the Reggeon field theory (RFT) [11].

This universality conjecture has been extended to include multicomponent systems [12] and systems with, an infinite number of absorbing states [13,14]. On the other hand, some other, less broad, universality classes of systems with absorbing states have been identified in recent years. They all include some extra symmetry or conservation law, foreign to the DP class. For example, if two symmetric absorbing states exist (which in many cases is equivalent to having activity parity conservation [15]), the universality class is other than DP, and the corresponding field theory differs from RFT [16]. A second example is constituted by systems with absorbing states in which fluctuations occur only at the interfaces separating active from absorbing regions, but not in the bulk of compact active regions (examples of this are the *voter model* or compact directed percolation [17]). In this case the exponents are also non-DP. A third and last example is that of systems with many absorbing states in which the activity field is coupled to an extra conserved field. This type of situation appears, for example, in conserved sandpile models, and has been recently shown to define a new universality class [8,18]. Apart from these and some few other well-known examples [19], systems with absorbing states belong generically into the DP universality class.

Recently, Lipowski has proposed a very simple, biologically motivated model, exhibiting a continuous transition into an absorbing phase, and claimed that this model shows a sort of ''superuniversality,'' i.e., in both one and two dimensions the model has the same critical exponents, namely, those of one-dimensional DP. Consequently, the system has been hypothesized to show a rather strange ''dimensional reduction'' [20] in two dimensions. This conclusion, if confirmed, would break the Grassberger-Janssen conjecture, since it is not clear that any new symmetry or extra conservation law is present in this model. In what follows we show the physical reasons why this model does not show directed percolation behavior: the presence of what we called *superabsorbing sites* is at the basis of this anomalous behavior. We will discuss also how DP can be restored by changing the geometry of the lattice on which the model is defined.

II. MODEL

The Lipowski model is defined operationally as follows: let us consider a square d-dimensional lattice. At a bond linking neighboring sites i,j, a random variable $w=w_{ij}$ is assigned. Different bonds are uncorrelated, and w is distributed homogeneously in the interval $[0,1]$. At each site i one defines r_i as the sum of the four bonds connecting this site to its four nearest neighbors. If r_i is larger than a certain threshold, r (that acts as a control parameter) the site is declared active, otherwise the site is inactive or absorbing. Active sites are considered unstable; at each step one of them is

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chosen randomly and its four associated w_{ij} bond variables are replaced by four freshly chosen independent random values (extracted from the same homogeneous probability distribution), and time is incremented by an amount of $\Delta t = 1/n(t)$, where $n(t)$ is the number of active sites at that time. Critical exponents are defined as it is customary in the realm of absorbing phase transitions [1].

It is clear that for small values of r, for instance $r=0$, the system will always be active, while for large enough values of r an absorbing configuration (with $r_i < r$ for all sites i) will be eventually reached. Separating these two regimes, we observe a critical value of r, r_c, signaling the presence of a continuous phase transition. In $d=1$, $r_c \approx 0.4409$ [23], while for $d=2$ we find $r_c = 1.386\,43(3)$. As bond variables are continuous, it is obvious that there is a continuous degeneracy of the absorbing state (i.e., infinitely many absorbing configurations).

In the one-dimensional case, all the measured critical exponents take the expected DP values [23], compatible with theoretical predictions for systems with many absorbing states [14,24]. The only discrepancy comes from the fact that the spreading exponents η and δ (see Sec. III B for definitions) appear to be nonuniversal, but the combination $\eta+\delta$ coincides with the DP expectation. This nonuniversality in the spreading is, however, generic of one-dimensional systems with an infinite number of absorbing states [25,24], and therefore it does not invalidate the conclusion that the system behaves as DP.

In two dimensions the only measured critical exponent in [26] is the order parameter one, β, which has been reported to take a value surprisingly close to the one-dimensional DP expectation $\beta \approx 0.27$ [26]. Based on this observation, Lipowski claimed that the system exhibits a sort of dimensional reduction. This possibility would be very interesting from a theoretical point of view and elucidating it constitutes the main original motivation of what follows.

Finally, let us mention that for spreading experiments it was found that, as happens generically in two-dimensional systems with many absorbing states [24,27], the critical point is shifted, and its location depends on the nature of the absorbing environment that the initial seed spreads in. In particular, the annular type of growth described in Ref. [26] in the case of spreading into a favorable media is typical of spreading in two-dimensional systems with many absorbing states, and it is well known to be described by dynamical percolation [24,27].

III. MODEL ANALYSIS

In order to obtain reliable estimations for β and determine other exponents, we have performed extensive Monte Carlo simulations in $d=2$ combined with finite-size scaling analysis, as well as properly defined spreading experiments.

A. Finite-size scaling analysis

We have considered a square lattice with linear dimension L ranging from 32 to 256. Averages are performed over a number of independent runs ranging from 10^2 to 10^5 depending on the distance to the critical point and on system size. The first magnitude we measure is the averaged density of active sites, $\rho(L,r,t)$, which for asymptotically large times converges to a stationary value $\rho(L,r)$. Observe that for small system sizes the system always reaches an absorbing configuration in finite time and therefore the only truly stationary state is $\rho=0$. In order to extrapolate the right asymptotic behavior in the active phase one has to determine $\rho(L,r)$ averaged over the runs that have not reached an absorbing configuration. A peculiarity of this system is that its convergence toward a well-defined stationary state is very slow, fluctuations around mean values are extremely persistent and, therefore, a huge number of runs is needed in order to obtain smooth evolution curves. Owing to this fact, numerical studies are rather costly from a computational point of view. The reasons underlying such anomalously long-lived fluctuations will be discussed in forthcoming sections. The maximum times considered are 8×10^5 Monte Carlo steps per spin; this is one order of magnitude larger than simulations presented in [26]. Near the critical point the relaxation times are very large (larger than 10^5) and, in order to compute stationary averages, transient effects have been cut off. We observe the presence of a continuous phase transition separating the active from the absorbing phase at a value of $r \approx 1.38$.

Assuming that finite-size scaling holds [28] in the vicinity of the critical point point r_c, we expect for values of $r < r_c$ (i.e., in the active phase)

$$\rho(L,r) \sim L^{-\beta/\nu_\perp} \mathcal{G}(L/\Delta^{-\nu_\perp}), \tag{1}$$

where $\Delta = |r - r_c|$. Right at the critical point, this corresponds to a straight line in a double logarithmic plot of $\rho(L,r)$ vs L. In Fig. 1 it can be seen that, in fact, we observe a straight line as a function of $\log_{10}(L)$ for $r = 1.38\,643(3)$ that constitutes our best estimation of r_c. This finite-size analysis allows us to determine r_c with much better precision than in the previous estimations [26]. From the slope of the previous log-log plot we measure $\beta/\nu_\perp = 0.57(2)$, which is quite far from both the one-dimensional DP exponent $\beta/\nu_\perp = 0.2520(1)$ and the two-dimensional value 0.795(5).

We have considered the larger available system size $L=256$ and studied the time decay of a fully active initial state for values of r close to r_c in the active phase (see Fig. 2). The stationary values for large values of t should scale as $\rho(L,r) \sim \Delta(L)^\beta$. From the best fit of our data (see Fig. 3) we determine both $r_c(L=256) \approx 1.386\,45$ and $\beta = 0.40(2)$. At the critical point, $\rho(r=r_c,t) \sim t^{-\theta}$. From the asymptotic slope of the curve for $r_c(L=256)$ in Fig. 2, we measure $\theta = 0.275(15)$. In this way, we have already determined three independent exponents. From these, using scaling laws, we can determine others, as for example $\nu_\perp = \beta/(\beta/\nu_\perp) = 0.69(9)$ (to be compared with the DP prediction 1.09 in $d=1$ and 0.733 in two dimensions [29]).

To further verify the consistency of our results, we have considered $\rho(L,r)$ computed for different values of r and L and assumed that $\rho(L,r)L^{\beta/\nu_\perp}$ depends on r and L through the combination $L^{1/\nu_\perp}\Delta$ [1]. In Fig. 4, we show the corresponding data collapse, which is rather good when the previously reported values of β and $\nu_\perp$ are used. In the inset, we verify that the data points are broadly scattered when one-dimensional DP exponent values are considered, showing that the dimensional reduction hypothesis is not valid. Data

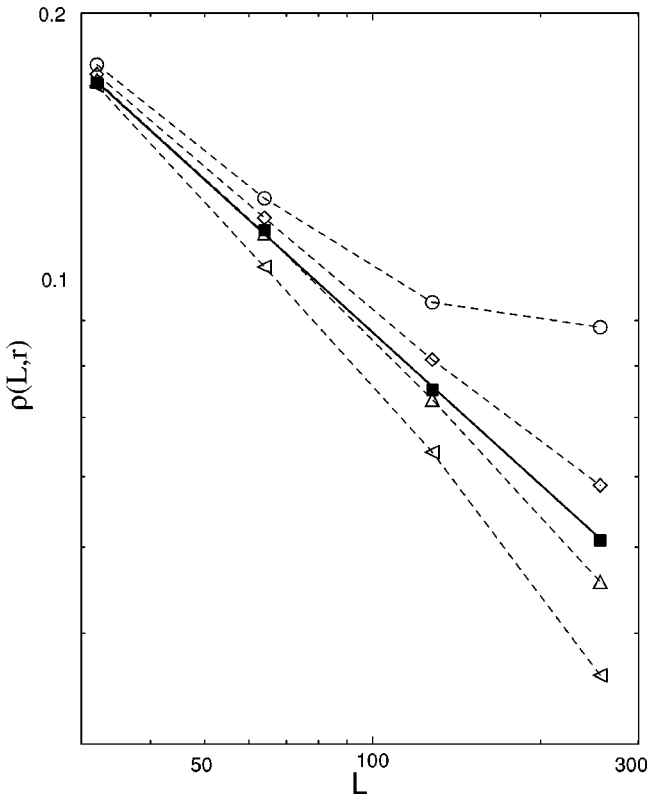

FIG. 1. Density of active sites as a function of L (the linear system size) for different values of r: from top to bottom, 1.386 30, 1.386 40, 1.386 43, 1.386 45, and 1.386 50, respectively. The straight solid line corresponds to the critical point $r_c = 1.386\,43(3)$.

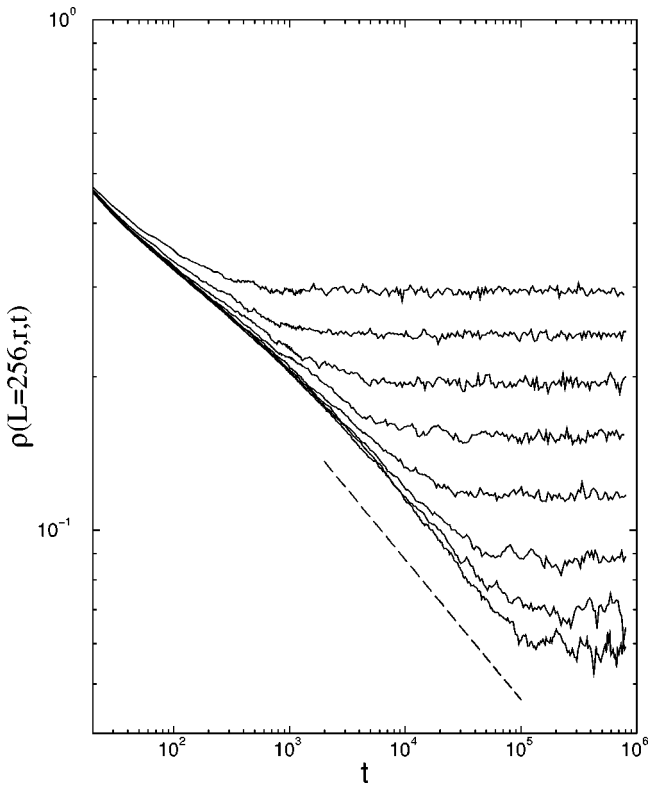

FIG. 2. Time evolution of the density of active sites for $L = 256$ and different values of r in the active phase, namely, from top to bottom $r = 1.381\,43$, 1.384 02, 1.385 27, 1.385 87, 1.386 16, 1.386 30, 1.386 37, and 1.386 40, respectively. From the slope of the straight dashed line we estimate $\theta = 0.275(15)$.

collapse is also not observed using two-dimensional DP exponents; this provides strong evidence that we are in the presence of anomalous (non-DP) scaling behavior. Finally, let us remark that the observed scaling does not extend over many decades for any of the computed steady-state magnitudes. Much better scaling is observed for spreading exponents as will be shown in the following section.

B. Spreading experiments and superabsorbing states

In order to further verify and support our previous conclusion, we have also performed spreading experiments as is customarily done in systems with absorbing states [30,1]. These consist of locating a seed of activity at the center of an otherwise absorbing configuration and studying how it spreads on average in that medium [1]. In the absorbing phase the seed dies exponentially fast, propagates indefinitely in the active phase, while the critical point corresponds to a marginal (power-law) propagation regime [1].

As stated before, it is well established that two-dimensional systems with an infinite number of many absorbing states show some peculiarities in studies of the spreading of a localized activity seed. The absorbing environment surrounding the seed can either favor or not favor the propagation of activity depending on its nature (see [24,27], and references therein). For the so-called *natural* initial conditions [1], the critical point for spreading coincides with the bulk critical point, and standard DP exponents are expected. In order to generate such natural configurations one could start the system with some highly active configuration and run the system right at the critical point; once it reaches an absorbing configuration it can be taken as a natural or self-generated environment for spreading. An alternative, more efficient way of proceeding, inspired in sandpile systems [8], is as follows. One considers an arbitrary absorbing configuration and runs a spreading experiment. Once the epidemic (or ''avalanche'' in the language of self-organized criticality [8]) is over, one considers the newly reached absorbing configuration as the initial state for a new spreading experiment avalanche. After iterating this process a number of times the system reaches a statistically stationary absorbing state; the natural one (see [8] and references therein). Using this absorbing state for spreading leads to DP exponent values (and critical point) in systems with many absorbing states as for example, the pair contact process [25,31].

By following this procedure, we have found a very peculiar property of this model that we believe to be at the basis of its deviating from DP. If the initial seed is located for all avalanches in the same site (or small group of localized sites), as is usually the case, after a relatively small number of avalanches the system reaches an absorbing configuration such that it is impossible to propagate activity for any possible forthcoming avalanche beyond a certain closed contour. For example, configurations as the one shown in Fig. 5(a) are generated. The four sites at the center are the ones at which activity seeds are placed in order to start avalanches. White sites are active and gray ones are absorbing. At each marked-in-black site, the sum of the three (black) bonds connecting it

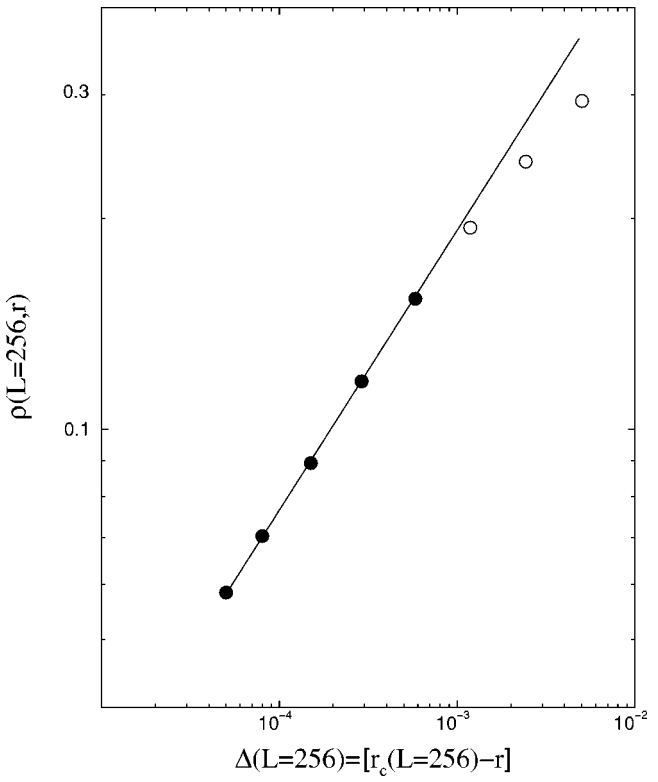

FIG. 3. Stationary density of active sites as a function of the distance to the critical point, for $L=256$ and different values of r in the active phase (the same values reported in Fig. 2). The best fit gives $\beta=0.40(2)$ and $r_c(L=256)\approx 1.386\,45$. Filled (empty) circles are used to represent scaling (not scaling) points.

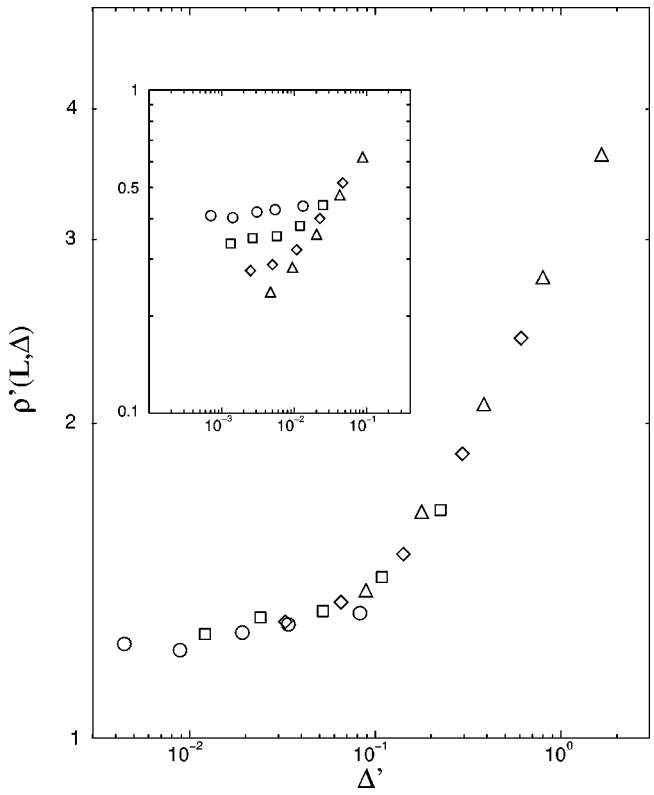

FIG. 4. Data collapse for the density of active sites: $\rho'(L,\Delta)=\rho(L,\Delta)L^{\beta/\nu_\perp}$ and $\Delta'=\Delta L^{1/\nu_\perp}$. Using the obtained exponent values $\beta/\nu_\perp\approx 0.57$ and $\nu_\perp\approx 0.69$, a reasonably good data collapse is observed. In the inset we show an attempt to collapse data using one-dimensional DP exponent values. There is no evidence of scaling neither in this case nor using two-dimensional DP exponents.

to sites other than a central one is smaller than $r_c-1=0.38\,643(3)$. In this way, regardless of the value of the bond connecting the site to the central region, the site remains inactive; it is a *superabsorbing site*. The existence of ''inactive forever'' sites has been already pointed out by Lipowski [26,32]. In the configuration shown in Fig. 5(a), activity cannot propagate out of the ''fence'' of superabsorbing sites; the cluster of superabsorbing sites will remain frozen indefinitely, and activity cannot possibly spread out. All avalanches will necessarily die after a few time steps. This type of blocking structure is quite generic and appears in all experiments after some relatively short transient. In conclusion, this way of iterating spreading experiments leads always to blocking closed configurations of superabsorbing sites instead of driving the system to a natural absorbing configuration.

Observe that some activity put out of a blocking fence of sites in Fig. 5(a) could well affect any of the external bonds of the superabsorbing sites [the dangling black bonds in Fig. 5(a)], converting the corresponding site to an absorbing or even an active one. Therefore, in order to overcome this difficulty of the frozen blocking configurations and be able to perform spreading experiments in some meaningful way, we iterate avalanches by locating the initial seed at randomly chosen sites in the lattice. In this way there is always a nonvanishing probability of destroying blocking ''fences'' by breaking them from outside as previously discussed. Measurements of the different relevant magnitudes are stopped when the system falls into an absorbing configuration or, alternatively, whenever a linear distance $L/2$ from the avalanche origin is reached. Observe that in the second case the dynamics has to be run farther in order to reach a new absorbing configuration at which to launch the next avalanche.

We monitor the following magnitudes: the total number of active sites in all the runs as a function of time $N(t)$ (we also estimate $N_s(t)$ defined as the average number of active sites restricted to surviving runs), the surviving probability $P(t)$, and the average square distance from the origin, $R^2(t)$. At the critical point these are expected to scale as $N(t)\sim t^{\eta}$, $P(t)\sim t^{-\delta}$ and $R^2(t)\sim t^{z}$. Results for these types of measurements are reported in Fig. 6. We obtain rather good algebraic behaviors at the previously estimated critical point r_c, confirming that the iteration-of-avalanches procedure leads the system to a natural absorbing environment. Slightly subcritical (supercritical) values of r generate downward (upward) curvatures in this plot for all the four magnitudes. Our best estimates for the exponents at criticality are: $z=0.96(1)$, $\eta=0.05(1)$, $\delta=0.66(1)$ (see Table I). To double check our results we also plot $N_s(t)$, which is expected to scale with an exponent $\eta+\delta$. An independent measurement of its slope in the log-log plot gives $\eta+\delta=0.71(1)$, in perfect agreement with the previously obtained results.

We can use these values to verify the hyperscaling relation [33,24]

$$\eta+\delta+\theta=\frac{dz}{2}. \tag{2}$$

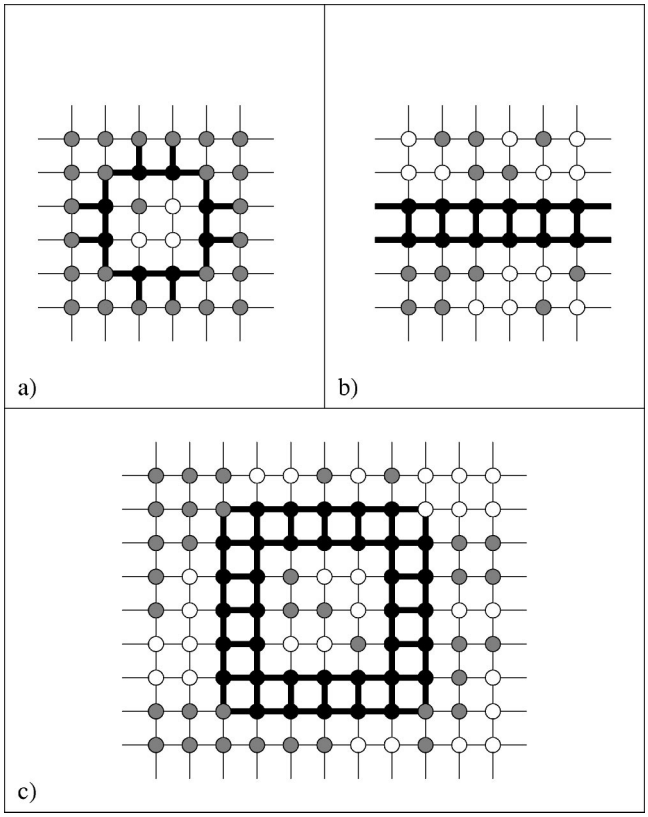

FIG. 5. Different frozen configurations of superabsorbing (black) sites. White (gray) color stands for active (absorbing) sites. (a) Blocking configuration for spreading from the central cluster of four sites. Black sites cannot change their state whatever the state or dynamics inside the cluster might be. Black bonds remain also frozen. (b) Spanning frozen cluster of superabsorbing sites. (c) Almost-frozen cluster of superabsorbing sites. This, and analogous structures, can be destabilized from the outside corners.

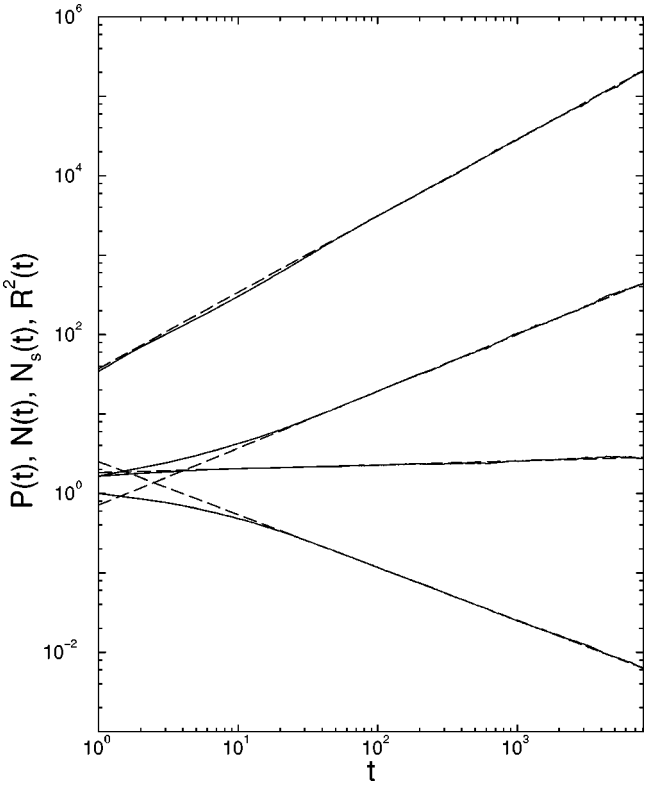

FIG. 6. Numerical results for spreading experiments. $R^2(t)$ (topmost curve), $N_s(t)$ (second curve from above), $N(t)$ (third curve from above), and $P(t)$ (bottom curve). From the slopes we estimate $z=0.96(1)$ and $\eta+\delta=0.71(1)$, $\eta=0.05(1)$, and $\delta=0.66(1)$, respectively.

Substituting the found values for z and $\eta+\delta$, we obtain $\theta \approx 0.25(2)$, compatible within error bars with the previously determined value $\theta=0.275(15)$.

One more check of the consistency of our results by using scaling laws is the following. As $z=2\nu_\perp/\nu_\parallel$ [29], we can estimate $\nu_\parallel$ from z and $\nu_\perp$. Then, using $\nu_\parallel$ and the fact that $\theta=\beta/\nu_\parallel$ we obtain $\theta=0.27(1)$, again in excellent agreement with the directly measured value.

In Table I, we present the collection of exponents and compare them with DP values in both one and two dimensions [29]. There is no trace of dimensional reduction; this model does not behave, at least up to the scales we have analyzed, as any other known universality class.

C. More about superabsorbing states

Let us recall our definition of superabsorbing states. A site, three of whose associated bonds take values such that the sum of them is smaller that $r-1$, cannot be activated from the remaining direction by neighboring activity. We say that this site is superabsorbing in that direction (or it is in a superabsorbing state). A site can be superabsorbing in one or more than one directions. Still, a site in a superabsorbing state can obviously be activated by neighboring activity in any of the remaining directions (if any).

Having stated the existence of frozen clusters in standard spreading experiments (when initialized from a fixed localized set of sites), one may wonder whether there are similar frozen structures in simulations started with an homogeneous initial distribution of activity, or in the modified type of spreading experiments we have just used (i.e., allowing the initial seed to land at a randomly chosen site) in the neighborhood of the critical point.

In principle, for any finite lattice, the answer to that question is affirmative. In Fig. 5(b) we show the shape of a frozen cluster of superabsorbing sites: any of the sites in it is superabsorbing with respect to the corresponding outward direction, and it cannot be ''infected'' from any of the other directions as neighboring sites are similarly superabsorbing. If a cluster like that is formed (or put by hand on the initial state) it will remain superabsorbing forever. However, the probability of forming such a perfectly regular chain is ex-

TABLE I. Exponent values for the two-dimensional Lipowski model and directed percolation in both one and two dimensions. Figures in parentheses denote statistical uncertainty (note that error bars are statistical errors coming from power-law fittings, and therefore do not include eventual systematic corrections to scaling).

Model	β	$\beta/\nu_\perp$	θ	η	δ	z
Lipowski	0.40(2)	0.57(2)	0.275(15)	0.05(1)	0.66(1)	0.96(1)
DP, $d=1$	0.276	0.252	0.159	0.313	0.159	1.265
DP, $d=2$	0.583	0.795	0.450	0.229	0.450	1.132

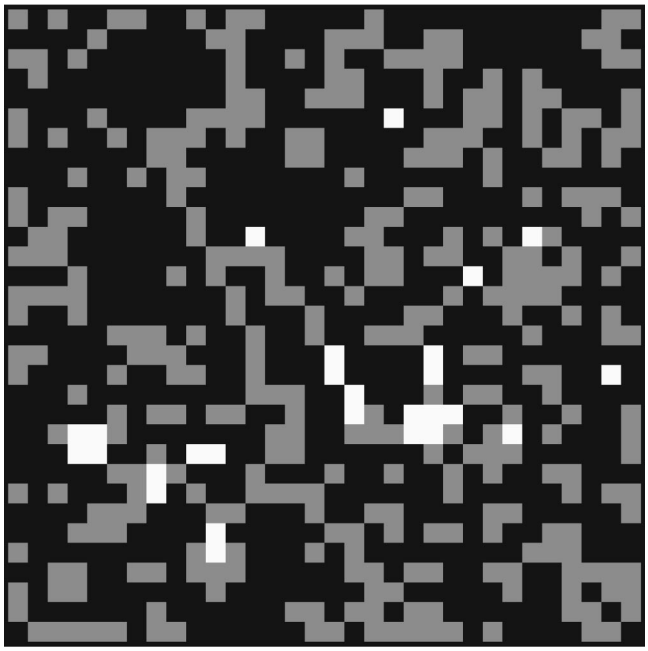

FIG. 7. Snapshot of a configuration in a 32×32 lattice in the stationary regime for a value of r close to the critical point. White color denotes activity, black corresponds to superabsorbing sites, while gray stands for absorbing sites. Observe that superabsorbing sites percolate through the lattice.

tremely small for large system sizes. Observe also that in order to have a completely frozen two-site broad-band structure it has to be unlimitedly long (or closed if periodic boundary conditions are employed). If instead it was finite, then sites at the corners would be linked to two external susceptible-to-change bonds and, therefore, these sites would be susceptible to become active because they would not be blocked forever. In this way any finite structure of superabsorbing sites in the square lattice is unstable. It can be eaten up (though very slowly) by the dynamics and is therefore not fully frozen. For instance, the cluster of superabsorbing sites represented in Fig. 5(c) is almost frozen but not really frozen as it may lose its superabsorbing character from the outside corners as previously described. Analogously, any other cluster shape of superabsorbing sites may be destabilized from its outside corners.

In conclusion, frozen clusters of superabsorbing sites do not appear spontaneously. However, almost-frozen regions do appear and may have extremely long life spans, especially close to the critical point where activity is scarce, and therefore the possibility of destabilizing them is small. In order to give an idea of how frequently superabsorbing sites appear, we present in Fig. 7 a snapshot of a typical system state near the critical point. White corresponds to active sites, while the remaining sites are absorbing: in black we represent superabsorbing (in one or more directions) sites, while simple absorbing (nonsuperabsorbing) sites are marked in gray color. Observe that superabsorbing sites are ubiquitous; in fact they percolate through the system. Among them, about one-fourth are superabsorbing in all four directions.

Even though none of the clusters of superabsorbing sites is completely frozen, and in principle, activity could reach any lattice site, the dynamics is *glassy* [34] in some sense. For instance, imagine an active region separated from an absorbing region by a line of superabsorbing-in-the-direction-of-the-activity sites. In order to reach the absorbing region, activity has to circumvent the superabsorbing barrier. But near the critical point, where activity is scarce, barriers of superabsorbing sites are intertwisted among them forming structures that, even if not completely frozen, are very unlikely to be infected. Activity has to overcome them progressively in order to reach the interior of superabsorbing regions. This resembles some aspects of glassy systems for which degrees of freedom are hierarchically coupled and, at observable timescales, they may appear effectively frozen [34].

This phenomenology is certainly very different from DP, and it is the reason why the relaxation toward stationary states is so slow, and why deviations from mean values are so persistent in numerical simulations. In particular, as superabsorbing regions are long-lived, the time required for the system to self-average is very large, and as near the critical point the probability of reaching an absorbing state is large, in practice, the system does not have the time to self-average. Consequently, a huge amount of independent initial states and runs have to be considered in order to measure smooth well-behaved physical magnitudes [35]. We strongly believe that this type of pathological dynamics is responsible for the departure of the Lipowski model from the DP universality class in two dimensions.

At this point one might wonder whether the one-dimensional version of this model is essentially different. Or in other words, why (one-dimensional) DP exponents are observed in $d=1$ [23]? The answer to this question is not difficult if one argues in terms of superabsorbing sites. First of all notice that in $d=2$, $r_c>1$. This means that just by changing one bond, whatever the value of the output is, the site can stay below threshold if the other three bonds sum less than r_c-1; this is to say superabsorbing states do exist at criticality. However in $d=1$, $r_c=0.4409<1$. In this case, by changing one bond value it is always possible to activate the corresponding site: superabsorbing sites do not exist in $d=1$ at the critical point [36]. Once the ''disturbing'' ingredient is removed from the model, we are back to the DP class as general principles dictate.

D. The honeycomb lattice

In order to further test our statement that superabsorbing states are responsible for the anomalous scaling of the two-dimensional Lipowski model, we have studied the following variation of it. We have considered the model defined on a honeycomb lattice (with three bonds per site), and performed Monte Carlo simulations. In this case there is the (geometrical) possibility of having completely frozen clusters of superabsorbing sites (see Fig. 8). The main geometrical difference from the previous case comes from the fact that here cluster corners are linked only to one external bond, and therefore are more prone to form frozen clusters. In principle, before performing any numerical analysis, there are two alternative possibilities: either the critical point is located at a value of r smaller than one or larger than one. In the first case, there would be no superabsorbing site (in analogy with the one-dimensional case); in the second case, pathologies associated with superabsorbing sites should be ob-

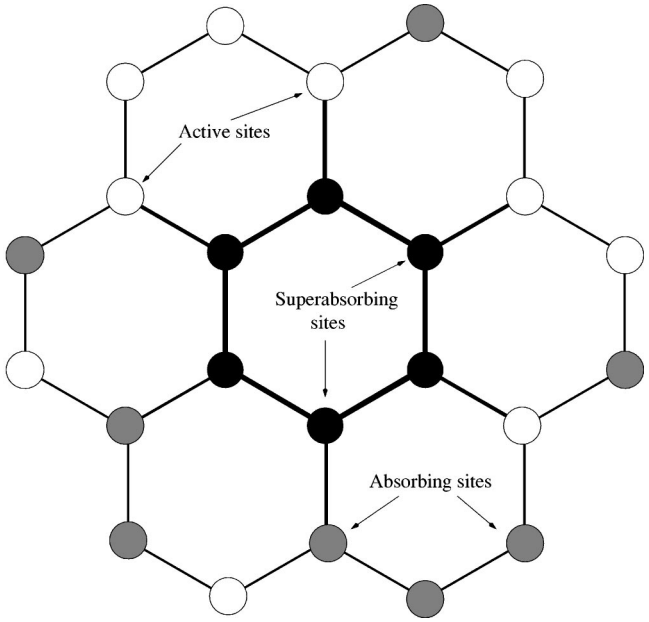

FIG. 8. Frozen cluster in the honeycomb lattice. This type of frozen structure of superabsorbing sites remains indefinitely superabsorbing at the critical point. Black: superabsorbing sites. Gray: absorbing sites. White: active sites.

served. The case $r_c=1$ would be marginal. Finite-size scaling analyses indicate the presence of a continuous phase transition located at $r\approx 1.0092$ (very nearby the marginal case, but significantly larger than $r=1$).

For Monte Carlo simulations, we have employed lattices of up to a maximum of 256×256 sites. All the observed phenomenology is perfectly compatible with two-dimensional DP behavior. The dynamics does not show any of the anomalies described for the square lattice case. In particular, from the dependence of the stationary activity density on system size we evaluate $\beta/\nu=0.80(1)$; from the time decay at criticality, $\theta=0.45(1)$, and finally $\beta=0.57(2)$; fully confirming consistency with two-dimensional DP behavior. This result seems to be in contradiction with the two alternative possibilities presented above. Let us now discuss why this is the case.

As the coordination number is three in this case, the sum of two bond values has to be smaller than $r_c-1\approx 0.0092$ in order to have a superabsorbing site in the direction of the remaining bond at criticality. As the two bonds are independent random variables, the probability of creating a superabsorbing site if the two of them are changed is fewer than 0.5% and the probability of generating frozen clusters (composed by six neighboring superabsorbing sites as shown in Fig. 8) is negligible at the critical point. In fact, we have not been able to observe any of them in our simulations. This means that one should study extremely large system sizes and extraordinarily long simulations in order to see anomalies associated with superabsorbing sites, otherwise, for any feasible simulation the behavior is expected to be DP-like. The observation of DP exponents in this case strongly supports the hypothesis that superabsorbing states are at the basis of the anomalous behavior of the model on the square lattice.

However, strictly speaking, the system should exhibit a (unobservable) first-order phase transition at $r=1$ in the thermodynamic limit. Indeed, for values of r larger than one there is a finite, though extremely small, probability of creating frozen clusters of superabsorbing sites (as the one in Fig. 8). As this is an irreversible process, after some (divergently long) transient there would be a percolating network of frozen clusters of superabsorbing sites, and the only possible stationary state would be an absorbing one with zero activity. On the other hand, for values of r smaller than unity, the probability of creating superabsorbing sites is strictly zero, and there will be a nonvanishing density of activity. As the density at $r=1$, almost independent of system size, is $\rho\approx 0.18$, the transition is expected to be discontinuous, and therefore the DP transition observed in our simulations is merely a finite-size effect, and should disappear for large enough sizes and long times. In any case, this first-order transition is unobservable computationally.

IV. SUMMARY

Summing up, we have shown that the two-dimensional Lipowski model does not belong to any known universality class. We have measured different critical exponents by running Monte Carlo simulations started from homogeneous initial states and also by performing spreading experiments. In any case, we find absolutely no trace of dimensional reduction, and neither is there evidence for the system to behave as two-dimensional DP. Instead, a different type of scaling behavior is observed. The main relevant physical ingredient of this class is the presence of superabsorbing sites, and almost-frozen clusters of superabsorbing sites that slow down enormously the dynamics.

The previous conclusion is strongly supported by two other observations: (i) the regular DP behavior observed in the one-dimensional version of the model for which superabsorbing states do not appear at criticality, and (ii) the two-dimensional DP behavior observed for the two-dimensional model defined on a honeycomb lattice, for which the probability of generating superabsorbing sites at criticality is almost negligible.

In general, superabsorbing sites can either arrange into completely frozen clusters or not depending on dimensionality, coordination number and other system details. Let us distinguish three main cases.

(1) When completely frozen clusters of superabsorbing sites appear below (or above) a certain value of the control parameter but not above (below), first-order transitions are expected (as occurs in the multiplicative model discussed in Appendix B [32]).

(2) If completely frozen clusters do not appear at criticality, but instead almost-frozen clusters are present, we expect anomalous behavior (as occurs in the original Lipowski model [26]).

(3) If neither frozen nor almost-frozen clusters are observed at criticality (as is the case for the one-dimensional version of the model [23]) we expect standard directed percolation behavior.

Two possible followups of this work are the following.

(1) It would be worth studying in more realistic situations as, for instance, in surface catalysis (dimer-dimer or dimer-

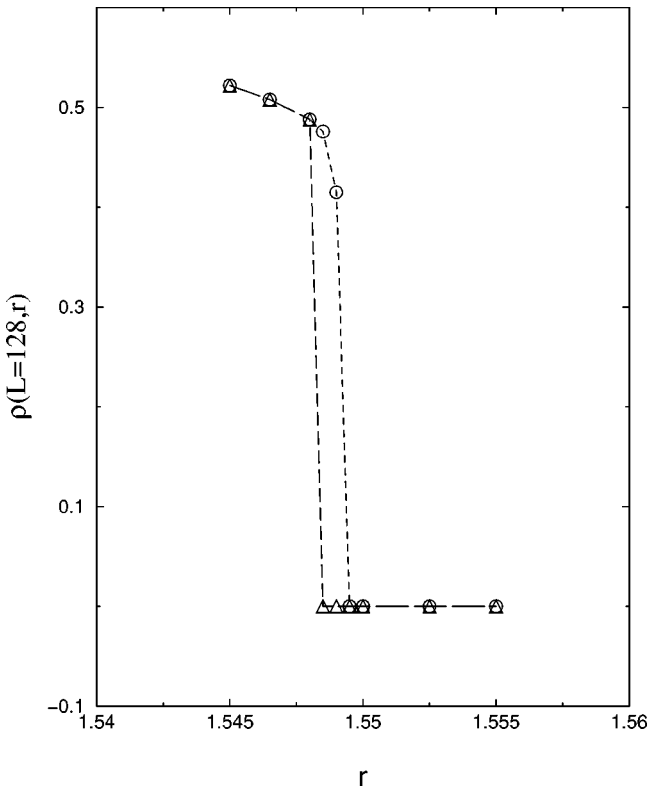

FIG. 9. Order parameter as a function of r in the case of parallel updating. The transition appears to be discontinuous in this case, exhibiting also a hysteresis loop.

trimer) models [13] whether effects similar to those described in this paper play any relevant role. In particular, for those models depending upon lattice and particle geometry there are cases in which activity cannot propagate to neighboring regions but is constrained to evolve following certain directions or paths. It would be rather interesting to sort out whether anomalies reported for those models [13] are related to the existence of superabsorbing states.

(2) From a more theoretical point of view, an interesting question is the following: What is the field theory or Langevin equation capturing the previously described phase transition with superabsorbing states? How does it change with respect to Reggeon field theory? Establishing what this theory looks like would clarify greatly at a field theoretical level the effect of superabsorbing states on phase transitions, and would permit to shed some light on the degree of universality of this anomalous phenomenology. Our guess is that a Reggeon field theory [11,10] with a spatiotemporal dependent anisotropic Laplacian term (which, for example, would enhance, not favor or forbid diffusion from certain sites in certain directions) could be a good candidate to describe this new phenomenology. Analogously to what happens in field-theoretical descriptions of other systems with many a absorbing states [13,24], the inhomogeneous Laplacian-term coefficient should be described by a second physical field coupled to the activity field in such a way that its fluctuations would vanish upon local absence of activity. Further pursuing this line of reasoning is beyond the scope of the present paper. As long as this program has not been completed, is not safe to conclude unambiguously that the anomalies described in this paper are relevant in the limit of extremely large times and system sizes.

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APPENDIX A

As an alternative attempt to speed up the dynamics, and examine further some properties of the two-dimensional model, we have implemented the microscopic dynamics replacing the original sequential updating by a synchronous or parallel one, i.e., all active sites are ''deactivated'' simultaneously at each Monte Carlo step, and all their associated bonds are replaced by new random variables simultaneously. In this way, as random numbers do not have to be extracted to sequentially select sites, the dynamics is largely accelerated. For this modified dynamics, we have examined some relatively large system sizes, $L=256$, and concluded that the nature of the transition is changed with respect to the sequential updating case. In this case the transition is first order and critical exponents cannot be defined. To show that this is the case, in Fig. 9 we present the stationary activity curve. The upper curve corresponds to simulations performed taking an initial activity density equal to unity. On the other hand, the lower curve is obtained by starting the system with a natural absorbing configuration, and activating on the top of it a small percentage of sites (about 10%).

For values of r in the interval $[\approx 1.545, \approx 1.555]$ the system reaches different states depending upon the initial condition. The presence of a hysteresis loop is a trait of the transition first-order nature. First-order absorbing-state transitions have been observed in other contexts [37]. However, we caution the reader that, as the transition is found to occur at a value of r for which the probability of creating superabsorbing sites is very large (much larger than in the sequential case), and the dynamics is therefore extremely anomalous and slow, it could be the case that the first-order character of the transition is only apparent. Extracting clean, conclusive results in the critical zone is a computationally very expensive task that we have not pursued.

APPENDIX B

Very recently, Lipowski has introduced a multiplicative version of his model on the square lattice in which sites are declared active if the product of the four adjacent bonds is smaller than a certain value of the control parameter r [32]. Bonds take uncorrelated values in the interval $[-0.5,0.5]$ extracted from a homogeneous distribution. For values of r smaller than $r=0$ there is a finite (not small) probability of generating superabsorbing sites. In this case, it is not difficult to see that isolated superabsorbing sites remain frozen forever. In analogy to the discussion of the honeycomb-lattice model, a first-order transition is expected at $r_c=0$ (as dis-

cussed also in [32]). However, in this case, as the probability of creating superabsorbing sites is not negligible, the first-order transition is actually observable. Based on a numerical measurement of β, Lipowski concludes that the model shares first-order properties with second-order features. In particular, the transition is clearly shown to be discontinuous, there is no diverging correlation length, but β is claimed to be, however, in the two-dimensional DP class. Our guess is that this apparent puzzle is simply due to a numerical coincidence and that in fact there is no trait of any second-order phase transition feature (observe that the fit for beta in [32] spans for less than half a decade in the abscise of the log-log plot).

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